NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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THEORETICAL ANALYSIS OF SOME SIMPLE TYPES OF ACCELERATION RESTRICTORS

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SUMMARY

A theoretical analysis has been made of acceleration restrictors which work on the principle of stopping the upward motion of the elevator when the signal from an acceleration-sensing device reaches a certain value. The devices considered for measuring the acceleration include an accelerometer located at the center of gravity, an accelerometer located 3 chords ahead of the center of gravity, and a device measuring the quantity: True airspeed times pitching velocity. The results of the analysis are presented as charts showing the ratio of peak acceleration to preset acceleration as a function of airspeed for various maximum rates of elevator movement. Calculations were made for a representative fighter airplane and a representative transport airplane over a range of center-of-gravity positions at altitudes of sea level and 40,000 feet.

The results of the analysis indicate that an acceleration restrictor sensing acceleration measured at the center of gravity is unsatisfactory because an undue limitation on the rate of control movement is required to prevent large overshoots of the acceleration beyond the preset limiting value. Somewhat larger rates of elevator movement are allowable if the accelerometer is located 3 chords ahead of the center of gravity. The allowable rate of elevator movement, however, is still insufficient to provide adequate maneuverability unless a device is incorporated to increase the maximum rate of elevator movement with decreasing speed. A device sensitive to a combination of the quantities normal acceleration and pitching acceleration might be suitable for operating an acceleration restrictor, but the component of pitching acceleration should be larger than that obtainable by simply locating the accelerometer in the nose of the airplane. A device sensitive to the quantity: True airspeed times pitching velocity appears to give fairly satisfactory acceleration-restriction characteristics provided the static margin is greater than about 10 or 15 percent of the mean aerodynamic chord.

An acceleration restrictor operated by a signal which precedes the build-up of normal acceleration by a sufficient amount will cause the elevator motion to stop at a deflection less than that corresponding to the preset acceleration. If the elevator motion is allowed to start again when the signal falls below the preset value, the elevator will move up in a series of steps and approach a deflection closely corresponding to the preset acceleration. This method of operation appears to offer promise as a means of avoiding excessive ratios of peak acceleration to preset acceleration over a wide range of flight conditions.

INTRODUCTION

Although existing airplanes are not equipped with acceleration restrictors, several recent developments have tended to create the need for a reliable method of limiting maneuvering accelerations. In the case of fighter airplanes the use of "g suits" allows the pilot to withstand physically accelerations in excess of the limit load factor of the airplanes. In the case of transport or bomber airplanes, research reported in reference 1 has shown that very low stick-force gradients are considered desirable provided the control friction is small. At the same time experience with control boosters has reached the point where control-force gradients as light as desired may be provided on airplanes of any size. In order to utilize such small control-force gradients safely, however, a device for preventing the pilot from inadvertently overloading the airplane in maneuvers would be required.

In connection with the design of any acceleration-restricting device, the question of reliability is of extreme importance. The device must work in a fool-proof manner on the rare occasions when it is required to prevent overloading the airplane and its presence must in no way endanger or interfere with the normal control of the airplane. Consideration must therefore be given to the use of the simplest possible devices even though such devices might operate satisfactorily over a more limited range of conditions than other more elaborate devices.

In the present paper, possible methods of operation of acceleration restrictors are discussed and an analysis is made of some of the simpler devices. The analysis is intended to supply information on the limitations of these devices and the conditions under which they might operate satisfactorily.

SYMBOLS

A	double amplitude of trapezoidal wave
A_N	coefficient of Fourier series
c	wing chord
C_m	pitching-moment coefficient $\left(\frac{M}{\frac{\rho}{2} V^2 Sc}\right)$
C_Z	vertical-force coefficient $\left(\frac{Z}{\frac{\rho}{2} V^2 S}\right)$
D	differential operator (d/ds)
g	acceleration due to gravity
k_y	radius of gyration about Y-axis
K_y	radius of gyration factor $\left(k_y/c\right)$
m	airplane mass
M	pitching moment (positive up)
n	normal acceleration (positive up), g-units
n_p	preset value of normal acceleration
q	pitching velocity
s	distance traveled, chords
S	wing area
t	time
T_f	period of trapezoidal wave, chords
T_1	duration of sloping portion of trapezoidal wave, chords
V	true airspeed

Z	vertical force (positive down)
l	distance between accelerometer and center of gravity, chords
α	angle of attack
δ_e	elevator angle
θ	angle of pitch
μ	relative-density factor $\left(m/\rho Sc\right)$
ρ	air density
ω	nondimensional frequency, radians/chord

Stability derivatives are defined in accordance with the following examples:

$$C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha}, \qquad C_{M_q} = \frac{\partial C_M}{\partial\left(\frac{qc}{2V}\right)}, \qquad C_{M_{D\alpha}} = \frac{\partial C_M}{\partial\left(\frac{\dot{\alpha}c}{2V}\right)}$$

POSSIBLE PRINCIPLES OF OPERATION OF ACCELERATION RESTRICTORS

Perhaps the simplest method for limiting maneuvering accelerations which shows enough promise to merit consideration is a device preventing further upward motion of the elevator when the limit load factor is reached. This type of system has the obvious shortcoming that in a maneuver the elevator may be rapidly moved to large deflections before the acceleration builds up. In order for the device to operate successfully, therefore, it is necessary to restrict the rate of elevator movement. Such a restriction is inherently provided in most control boosters; this provision, therefore, does not add to the mechanical complication of the device. An undue restriction in the rate of elevator movement may be required, however, in order to prevent the acceleration in a maneuver from exceeding the desired value. In the limiting case where the maneuvering stability is zero, a device which operates on the principle of stopping the upward elevator movement is obviously useless because any small upward movement of the elevator would result in excessive values of acceleration. Devices of this type, therefore, fail to provide safety in case the airplane is inadvertently loaded with an excessively rearward center-of-gravity location, just the condition under which limitation of the accelerations might be most needed. Such devices might, nevertheless, have application to airplanes whose physical arrangement makes abnormally tail-heavy loading unlikely, or to transport airplanes in which close control of the center of gravity is maintained.

Two methods may be employed to increase the allowable rate of control movement for satisfactory operation of an acceleration restrictor of the type described. One method is to operate the device in accordance with a signal which measures the normal acceleration under steady conditions but which precedes the build-up of normal acceleration in a rapid maneuver. The other method is to reduce the lag in airplane response to control movement.

In connection with the first method, a signal consisting of a combination of pitching acceleration and normal acceleration might be used to operate the acceleration restrictor. In this case, the pitching-acceleration signal precedes the normal acceleration during the early stages of a pull-up but later goes to zero in a steady pull-up and leaves only the normal-acceleration signal. Such a combined signal might be very easily obtained by locating the accelerometer some distance ahead of the center of gravity. Another source of a signal which precedes the normal acceleration in a rapid maneuver is a device measuring the quantity: True airspeed times pitching velocity. This quantity measures the steady geometric acceleration but fails to include the effect of gravity on the loads applied to the airplane. If this type of device were set to limit the acceleration to a safe value in a level attitude, therefore, it would provide a conservative limitation for other attitudes.

The second method for increasing the allowable rate of control movement, which involves reduction of the lag in airplane response, can be accomplished by incorporating a suitable automatic pilot which moves the elevator in accordance with a command signal from the control stick in such a way as to produce the acceleration called for with as little lag as possible. By use of an autopilot which increases the stability of the airplane, such an arrangement might be made to work even when the airplane without automatic control had zero stability in maneuvers. This method requires continuous operation in normal flight of the mechanisms connected with the restricting device, however, and therefore suffers from the objection that malfunctioning of the device might endanger the normal control of the airplane.

The present study has been confined to acceleration restrictors of the simplest type, that is, those which prevent further upward movement of the elevator when the signal from the acceleration-measuring device reaches a certain value. The study is considered preliminary because no general analysis has been made to determine the optimum design for such a device in any particular case. The devices considered for measuring the acceleration include an accelerometer located at the center of gravity, an accelerometer 3 chords ahead of the center of gravity, and a device measuring the quantity: True airspeed times pitching velocity. Calculations have been made for a representative fighter airplane and a representative transport airplane over a range of values of altitude, airspeed, and center-of-gravity position. No consideration has been given in the present paper to the mechanical design of a device to stop the elevator movement, though such a device should not be difficult to develop.

ANALYSIS

The most critical type of maneuver for an acceleration restrictor of the type considered is a pull-up in which the elevator is moved at the maximum available rate until it is stopped by the action of the restrictor and then is held fixed at this deflection. The effectiveness of the device may be determined by comparing the maximum normal acceleration reached with the desired limiting value.

Calculation of Response

In order to determine the response of an airplane in a maneuver in which the elevator is moved at a constant rate and thereafter held fixed, the response was actually calculated to a periodic elevator motion of trapezoidal wave form. The fundamental frequency of the wave was selected so that the response of the airplane had essentially reached a steady value before the next half-cycle started. The first step in this calculation was to determine the frequency-response characteristics of the airplane (that is, the response to sinusoidal elevator movements of various frequencies). The response to a periodic elevator motion of trapezoidal wave form was then calculated by expressing the trapezoidal wave as a Fourier series and calculating the response to the various terms of the series separately. These responses were added vectorially by an electromechanical Fourier synthesizer of the type described in reference 2. This machine allowed the inclusion of the first 23 harmonics of the Fourier series.

Maneuvers involving many combinations of elevator rate, elevator deflection, and airspeed may occur in practice. By employing the equations for response in nondimensional form, however, a relatively small number of solutions may be made to apply to a wide range of conditions. The detailed procedure used in the analysis is now described.

The transfer functions relating the normal acceleration and pitching velocity of the airplane to the elevator deflection are expressed in nondimensional form as follows:

$$\frac{D(\alpha - \theta)}{\delta_e} = \frac{D^2\left(-2\mu K_Y{}^2 C_{Z_{\delta_e}}\right) + C_{m_\alpha} C_{Z_{\delta_e}} - C_{Z_\alpha} C_{m_{\delta_e}}}{\Delta_o}$$

$$\frac{D\theta}{\delta_e} = \frac{D\left(-2\mu C_{m_{\delta_e}}\right) + C_{Z_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{Z_{\delta_e}}}{\Delta_o}$$

where

$$\Delta_o = D^2\left(-4\mu^2 K_Y{}^2 + \mu K_Y{}^2 C_{Z_{D\alpha}}\right) + D\left(2\mu K_Y{}^2 C_{Z_\alpha} + \mu C_{m_q} + \mu C_{m_{D\alpha}}\right) +$$

$$2\mu C_{m_\alpha} + \frac{1}{2} C_{Z_q} C_{m_\alpha} - \frac{1}{2} C_{m_q} C_{Z_\alpha}$$

The method of derivation of these equations is given in reference 3. The equations are derived on the assumption that the airspeed is constant. The nondimensional time is taken as the distance traveled expressed in chords. The normal acceleration in g-units is related to $D(\alpha - \theta)$ by the formula:

$$n = -\frac{V^2}{gc} D(\alpha - \theta)$$

The pitching velocity is related to $D\theta$ by the formula:

$$q = \frac{V}{c} D\theta$$

The transfer function for the normal acceleration at a location l chords ahead of the center of gravity is obtained from the relation

$$\left[\frac{D(\alpha - \theta)}{\delta_e}\right]_l = \frac{D(\alpha - \theta)}{\delta_e} - l \frac{D^2\theta}{\delta_e}$$

Frequency-response characteristics were computed by substituting $D = i\omega$ into the transfer functions. The real and imaginary parts of the resulting expression give the components of response in phase and 90° out of phase, respectively, with a sinusoidal elevator motion of frequency ω.

The symbols describing the trapezoidal wave form assumed for the elevator motion are illustrated in figure 1. This wave form may be expressed as a Fourier series

$$\delta_e = A_N \sin N\omega s$$

where

$$A_N = \frac{2A}{\pi N} \frac{\sin \pi N \frac{T_1}{T_f}}{\pi N \frac{T_1}{T_f}} \qquad (N = 1, 3, 5 \ldots)$$

Values of T_1/T_f of 0, 1/48, 1/24, 1/12, 1/6, and 1/3 were used in the calculations. The corresponding input waves as plotted by the Fourier synthesizer using values of N through 23 are shown in figure 2. It is seen that the square wave is rather poorly approximated by the assumed series, but the waves for values of T_1/T_f of 1/24 or more are well-approximated. These waves correspond to the conditions of greatest interest in the present analysis.

Analysis of Response Curves

A typical response time history, obtained from the Fourier synthesizer in the manner described previously, is shown in figure 3. This figure is used to illustrate the method of application of the response curves to the determination of the action of an acceleration restrictor in any particular maneuver. Scales are not shown in the curves of figure 3 because only the shape of the curves is important at this stage of the analysis. The magnitudes of the various quantities are to be determined from a knowledge of the airspeed, desired limiting acceleration, and so forth. Throughout the analysis, the response curves were assumed to start from a value of the variable of zero, so that any numerical value of acceleration represents an increment from the steady flight condition of 1 g. Similarly, all ratios of accelerations or pitching velocities represent ratios of the incremental values of these quantities.

Acceleration restrictor sensitive to normal acceleration measured at center of gravity.- First, the action of an acceleration restrictor which stops the upward motion of the elevator at a preset value of normal acceleration n_p measured at the center of gravity will be considered. Since, in figure 3, the upward motion of the elevator stops at time (1), the value of normal acceleration at this time n_1 represents this preset value of acceleration. The ratio of the peak acceleration to the preset acceleration is given by the ratio n_2/n_1. This ratio must be determined as a function of the rate of elevator motion and the airspeed. As shown in figure 3, the rate of elevator motion is δ_{e_1}/t_1. This value of δ_{e_1} is determined as follows:

$$\delta_{e_1} = \delta_{e_3} = n_3 \left(\frac{d\delta_e}{dn}\right)_{steady}$$

where $(d\delta_e/dn)_{steady}$ is the ratio of elevator angle to normal acceleration under steady conditions. Now

$$\left(\frac{d\delta_e}{dn}\right)_{steady} = -\frac{1}{\frac{V^2}{gc}\left[\frac{D(\alpha - \theta)}{\delta_e}\right]_{steady}}$$

where $\left[\frac{D(\alpha - \theta)}{\delta_e}\right]_{steady}$ is the known value of the nondimensional acceleration response at zero frequency. Also,

$$n_3 = n_1\left(\frac{n_3}{n_1}\right) = n_p\left(\frac{n_3}{n_1}\right)$$

and the value of t_1 is $(T_1/T_f)T_f c/V$, where T_f is the known fundamental period of the trapezoidal wave expressed in chords.

By use of the preceding relations, the expression for the rate of elevator movement becomes

$$\frac{\delta_e}{t_1} = \frac{gn_p\left(\frac{n_3}{n_1}\right)}{VT_f\left(\frac{T_1}{T_f}\right)\left[\frac{D(\alpha - \theta)}{\delta_e}\right]_{steady}}$$

This formula establishes the relation between the elevator rate and the airspeed for a given set of the values of the ratios n_3/n_1 and T_1/T_f obtained from a given nondimensional response time history. From this same time history a certain value of the ratio of peak acceleration to preset acceleration n_2/n_1 is obtained. This information allows the plotting of a contour line of constant ratio of peak acceleration to preset acceleration on a graph of elevator rate against airspeed. By interpolation between a series of such curves, a graph showing this ratio as a function of airspeed for constant values of elevator rate may be prepared. Since the chord c cancels in the formula, the same curves apply to dynamically similar airplanes of any size.

<u>Acceleration restrictor sensitive to the quantity: True airspeed times pitching velocity</u>.- The action of an acceleration restrictor which stops the upward motion of the elevator at a given value of the quantity Vq/g is now considered. (The g is placed in the denominator of this expression to put the numerical values of this quantity on the same basis as the values of n.) The value of q at time (1) (fig. 3) is the value required to make the quantity Vq/g equal to the preset value of normal acceleration n_p. The ratio of peak acceleration to preset acceleration is therefore $\frac{n_2}{Vq_1/g}$. But

$$q_1 = q_3 \left(\frac{q_1}{q_3}\right)$$

and in a steady pull-up,

$$\frac{Vq_3}{g} = n_3$$

Hence, the ratio of peak acceleration to preset acceleration is $\frac{n_2}{n_3}\frac{q_3}{q_1}$.

By the same method as was used previously for the accelerometer-actuated restrictor, the expression for the rate of elevator movement may be shown to be

$$\frac{\delta_e}{t_1} = \frac{gn_p \frac{q_3}{q_1}}{VT_f \frac{T_1}{T_f} \left[\frac{D(\alpha - \theta)}{\delta_e}\right]_{steady}}$$

<u>Acceleration restrictor sensitive to normal acceleration 3 chords ahead of the center of gravity</u>.- The method of analysis used for the case where the acceleration was measured at a point 3 chords ahead of the center of gravity is similar to that described for the device

sensitive to Vq/g. The formulas used to determine the rate of elevator movement and the ratio of peak acceleration to preset acceleration are identical in the two cases, except that values read from the curve of the acceleration at the location of the accelerometer are used in place of the values of q.

RESULTS

Calculations were made of the time histories of normal acceleration and pitching velocity in pull-ups with various rates of elevator movement for two airplanes, a representative fighter airplane and a representative transport airplane. These calculations were made for two altitudes, sea level and 40,000 feet, and three center-of-gravity positions, corresponding to values of static margin of 0, 10, and 20 percent of the mean aerodynamic chord. These cases were considered to be representative of the range of dynamic stability characteristics likely to be encountered in practice. The assumed airplane dimensions, mass characteristics, and stability derivatives are listed in table I.

In order to show the range of dynamic stability characteristics included in the analysis, typical examples of response curves for the fighter and transport airplanes at altitudes of sea level and 40,000 feet and with three center-of-gravity positions are shown in figure 4. The response to a rather rapid elevator movement, rather than to a theoretical step function, is shown because, as noted previously, the Fourier synthesizer more accurately approximated the case where the rate of elevator movement was finite. The period and damping of the short-period oscillation of the airplanes under these conditions are given in table II.

The behavior of the acceleration restrictors discussed previously is given in figures 5, 6, and 7. Figure 5 shows, for all the airplane conditions considered, the characteristics for the case in which the upward motion of the elevator is stopped at a given value of normal acceleration measured at the center of gravity. Figure 6 presents similar results for the case in which the acceleration is measured at a point 3 chords ahead of the center of gravity. These calculations are limited to the case of the fighter airplane at sea level. Figure 7 shows, for all the airplane conditions considered, the characteristics of an acceleration restrictor which stops the upward motion of the elevator at a given value of the quantity Vq/g. Each of these figures presents the ratio of peak acceleration to preset acceleration as a function of true airspeed for various values of elevator rate. The preset incremental normal acceleration was taken as 6g for the fighter airplane and 2.5g for the transport airplane. These curves may be applied to other values of preset acceleration, however, by changing the values of elevator rate in direct proportion to the value of preset incremental acceleration. The curves apply quantitatively to airplanes of any size which are dynamically similar to the airplanes whose characteristics are listed in table I.

The data of figures 5, 6, and 7 are plotted for values of static margin of 0, 10, and 20 percent of the mean aerodynamic chord. Inasmuch as the maneuver margin has more significance in interpreting these results, however, values of maneuver margin for the various cases are also shown in these figures. (Maneuver margin is defined as the distance between the center of gravity and the maneuver point; the maneuver point is the center-of-gravity location at which the variation of pitching moment with lift coefficient is zero in steady pull-ups at constant airspeed.)

DISCUSSION

Interpretation of the results shown in figures 5, 6, and 7 requires consideration of the desired action of an acceleration restrictor. Ideally, the acceleration reached in a pull-up should equal but should not exceed the preset limiting value. An overshoot of the acceleration beyond the preset value could be allowed for if the ratio of peak acceleration to preset acceleration were always the same. Unfortunately, however, the results show that this ratio usually increases approximately linearly with airspeed. If the preset acceleration is chosen to avoid exceeding the limit load factor in high-speed flight, therefore, the maneuvering capabilities of the airplane will be restricted to values below the structural limitations at lower airspeeds. In order to avoid an undue limitation in the maneuvering capabilities, the ratio of peak acceleration to preset acceleration must be kept to a fairly low value, say 1.5. Values of this ratio as high as 4 are shown in figures 5, 6, and 7. The values between approximately 1.5 and 4 are beyond the range of practical interest and are included simply to show the theoretical limitations of the devices.

The data of figures 5, 6, and 7 show, as expected, a strong effect of the maneuvering stability on the behavior of the acceleration restrictors. A maneuver margin of zero would result in infinite ratios of maximum acceleration to preset acceleration for any elevator rate for all of the acceleration restrictors considered. The detailed results for the various cases are now discussed.

<u>Acceleration restrictor sensitive to normal acceleration at the center of gravity</u>.- In the case of the acceleration restrictor which stops the elevator movement at a given value of normal acceleration measured at the center of gravity (fig. 5), the characteristics with a static margin of zero are entirely unacceptable, and relatively low values of elevator rates are required to avoid overshoot ratios greater

than 1.5 when the static margin is 10 percent or even 20 percent of the mean aerodynamic chord. For example, in the case of the fighter airplane at sea level (fig. 5(a)) with a static margin of 10 percent of the mean aerodynamic chord, the elevator rate would have to be restricted to about 6° per second for an overshoot ratio of 1.5 at an airspeed of 900 feet per second. Such a rate is probably unacceptably low inasmuch as a rate of elevator movement of about 30° to 50° per second is considered necessary for low-speed maneuvers such as landing in an airplane of this type. At an altitude of 40,000 feet (fig. 5(b)) somewhat higher rates are permissible, primarily because the airplane must pull up to a higher lift coefficient to obtain a given acceleration at high altitude.

The explanation for the large ratios of peak acceleration to preset acceleration with the higher rates of elevator movement is illustrated in figure 3. The main reason for the overshoot is not the dynamic overshoot of the acceleration $(n_2/n_3$ in fig. 3) but the lag in build-up of the acceleration which allows the elevator to be moved to a deflection far beyond that required for the desired limiting acceleration before the acceleration has approached its maximum value.

In the case of the transport airplane (figs. 5(c) and 5(d)) similar conclusions may be obtained. The results of research reported in reference 1 indicate that values of rate of elevator movement as low as 10° per second might be acceptable for the landing maneuver of a transport airplane. The values of elevator rate required to prevent overshoot ratios greater than 1.5 are considerably lower than this figure, however, at high values of airspeed.

Acceleration restrictor sensitive to normal acceleration 3 chords ahead of the center of gravity.- The results for the case of an acceleration restrictor depending on measurements of acceleration at a point 3 chords ahead of the center of gravity for the fighter airplane at sea level (fig. 6) show some increase in the allowable rates of elevator motion when compared with the results when the accelerometer was located at the center of gravity (fig. 5(a)). The increase is not sufficient, however, to result in satisfactory characteristics, inasmuch as the maximum allowable values of elevator rate at high values of airspeed are again less than those required in low-speed flight. The improved results obtained with the accelerometer mounted near the nose as compared to those with it mounted at the center of gravity indicate that a combination of the quantities normal acceleration and pitching acceleration might provide a suitable signal for operation of an acceleration restrictor but the relative contribution of the pitching acceleration obtainable by simply locating the accelerometer near the nose of the airplane is insufficient.

One way in which satisfactory characteristics might be obtained with an acceleration restrictor sensitive to normal acceleration measured at a point near the nose of the airplane would be to incorporate a device to increase the maximum rate of elevator motion as the airspeed decreased. Such a variation in rate of elevator motion with airspeed would probably be acceptable from the pilots' standpoint inasmuch as smaller rates of elevator motion are required in maneuvers at high speed. The complication involved in providing this characteristic would probably rule it out of consideration unless some other benefit could also be derived. One such benefit which would result is a reduction in the tail loads in maneuvers at high speed.

Acceleration restrictor sensitive to the quantity: True airspeed times pitching velocity.- The results obtained with an acceleration restrictor which stops the motion of the elevator at a given value of quantity Vq/g (fig. 7) indicate that with this device much higher rates of elevator motion are allowable. The characteristics of this device appear to be satisfactory for the case of the fighter airplane provided the static margin is greater than 10 percent of the mean aerodynamic chord. In the case of the transport airplane, however, somewhat greater values of static margin are required.

The ratio of peak acceleration to preset limiting acceleration for this type of device is seen to be less than 1.0 for the fighter airplane with low rates of elevator movement. This condition may not be undesirable provided the acceleration restrictor is designed to allow further upward movement of the elevator when the quantity Vq/g falls below the preset value. The characteristics obtained in this case are shown in figure 8. In a rapid maneuver the device stops the upward motion of the elevator at the preset value of the quantity Vq/g. Because the pitching velocity precedes the normal acceleration, the elevator stops at a point corresponding to a value of normal acceleration below the desired limiting value. Following the initial overshoot of pitching velocity, however, the quantity Vq/g falls below the preset value and further upward motion of the elevator is allowed. The elevator therefore moves up in a series of steps. In the case shown, the elevator motion ceased after three steps and the ratio of peak acceleration to preset acceleration was 1.23. Some oscillations in the response were induced by the motion of the elevator, but the oscillations in normal acceleration do not appear large enough to have a significant effect on the satisfactory operation of the airplane.

The foregoing type of behavior would be expected for any acceleration restrictor operated by a signal which precedes the build-up of normal acceleration by a sufficient amount. For example, an acceleration restrictor sensitive to a combination of pitching acceleration and normal acceleration would be expected to work in this manner provided the pitching-acceleration signal were sufficiently large. This method of operation appears to offer promise as a means for limiting the overshoot of the normal acceleration beyond its preset value over a wide range of flight conditions. Further investigation of devices working on this principle would therefore appear desirable.

CONCLUDING REMARKS

An acceleration restrictor which works on the principle of stopping the upward motion of the elevator at a given value of normal acceleration measured at the center of gravity is unsatisfactory for the representative fighter airplane and representative transport airplane investigated because an undue limitation on the rate of elevator movement is required to prevent large overshoots of the acceleration beyond the preset limiting value. Somewhat larger rates of elevator movement are allowable if the accelerometer is located 3 chords ahead of the center of gravity, because the component of pitching acceleration measured by the instrument precedes the normal acceleration in a rapid maneuver. The allowable rate of elevator movement, however, is still insufficient to provide adequate maneuverability unless a device is incorporated to increase the maximum rate of elevator movement with decreasing speed. These results indicate that a device sensitive to a combination of the quantities normal acceleration and pitching acceleration might be suitable for operating an acceleration restrictor but that the component of pitching acceleration should be larger than that obtainable by simply locating the accelerometer in the nose of the airplane.

The use of a device sensitive to the quantity: True airspeed times pitching velocity to operate the acceleration restrictor provides anticipation of the acceleration in a rapid maneuver. The allowable rates of elevator motion for the fighter and transport airplanes investigated appear satisfactory with this method provided the static margin is greater than 10 or 15 percent of the mean aerodynamic chord. This method does not take into account the effect of gravity on the loads experienced by the airplane. If the device were set to limit the acceleration to a safe value in level flight, therefore, it would provide a greater restriction than necessary in other attitudes.

An acceleration restrictor operated by a signal which precedes the build-up of normal acceleration by a sufficient amount will cause the elevator motion to stop at a deflection less than that corresponding to the preset acceleration. If the elevator motion is allowed to start again when the signal falls below the preset value, the elevator will move back in a series of steps and approach a deflection closely corresponding to the preset acceleration. This method of operation appears to offer promise as a means of avoiding excessive ratios of peak acceleration to preset acceleration over a wide range of flight conditions.

Langley Aeronautical Laboratory
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TABLE I

CHARACTERISTICS OF AIRPLANES USED IN CALCULATIONS

	Fighter	Transport
Weight, lb	15,000	22,000
Wing area, sq ft	300	990
Horizontal tail area, sq ft	60	180
Wing chord, ft	7	9
Tail length, ft	20	37
Radius of gyration about Y-axis, ft	7	12.33
μ (sea level)	93.3	32.2
μ (40,000 feet)	382	132
K_y	1.0	1.37
C_{Z_α}	-4.77	-5.50
$C_{Z_{D\alpha}}$	-2.12	-2.34
C_{Z_q}	-4.24	-4.72
$C_{Z_{\delta_e}}$	-0.37	-0.286
$C_{m_{D\alpha}}$	-6.04	-9.70
C_{m_q}	-12.06	-19.40
$C_{m_{\delta_e}}$	-1.05	-1.18

TABLE II

PERIOD AND DAMPING OF THE SHORT-PERIOD LONGITUDINAL OSCILLATIONS

OF THE AIRPLANES USED IN CALCULATIONS

Static margin (percent M.A.C.)	Fighter at -				Transport at -			
	Sea level		40,000 feet		Sea level		40,000 feet	
	P (chords)	$T_{1/2}$ (chords)	P (chords)	$T_{1/2}$ (chords)	P (chords)	$T_{1/2}$ (chords)	P (chords)	$T_{1/2}$ (chords)
0	-----	11.5 51.0	-----	47.2 205	-----	4.33 16.5	-----	17.5 67.9
10	141	18.9	259	76.7	222	6.86	217	27.8
20	93.8	18.9	180.5	76.7	87.9	6.86	145	27.8

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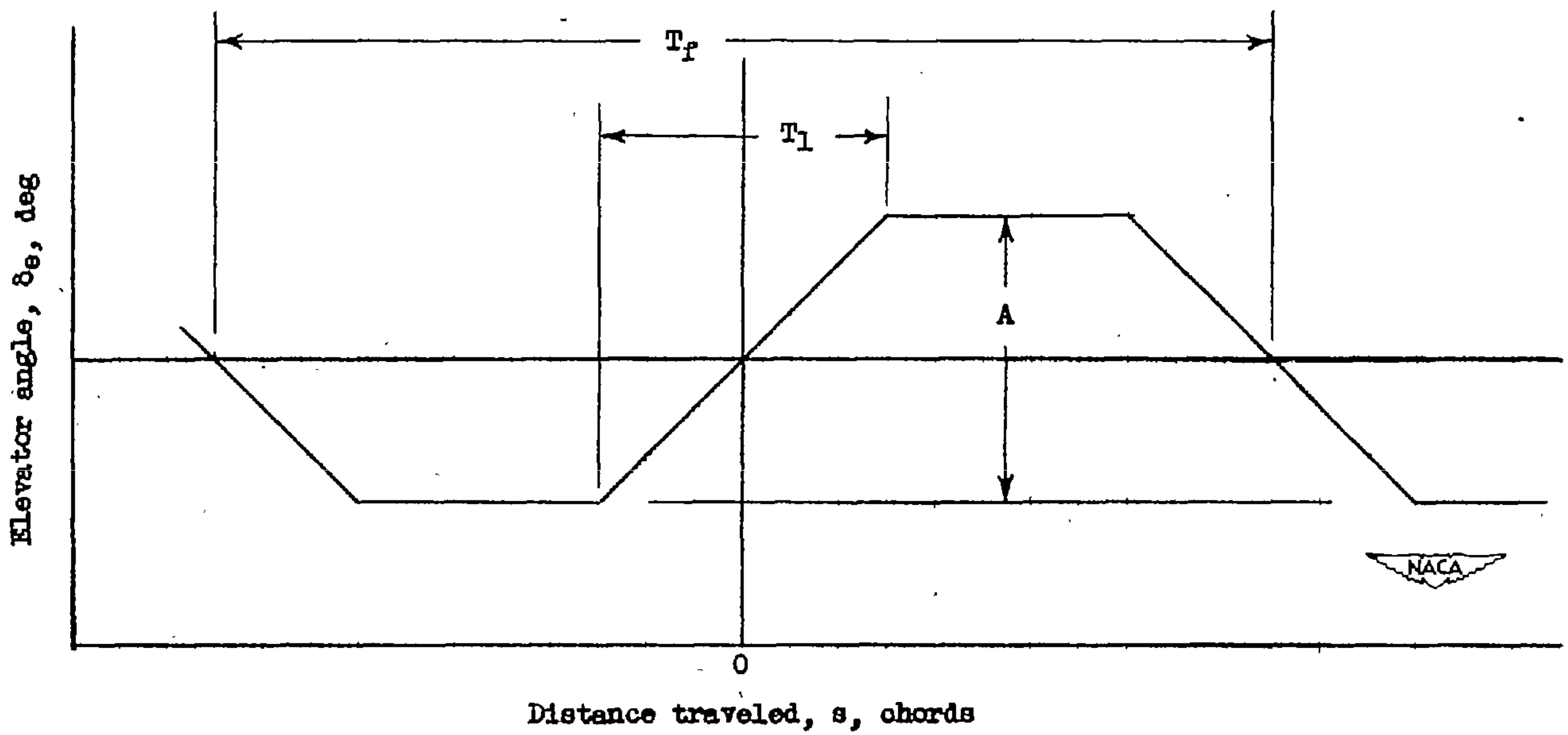


Figure 1.- Trapezoidal wave form of elevator motion.

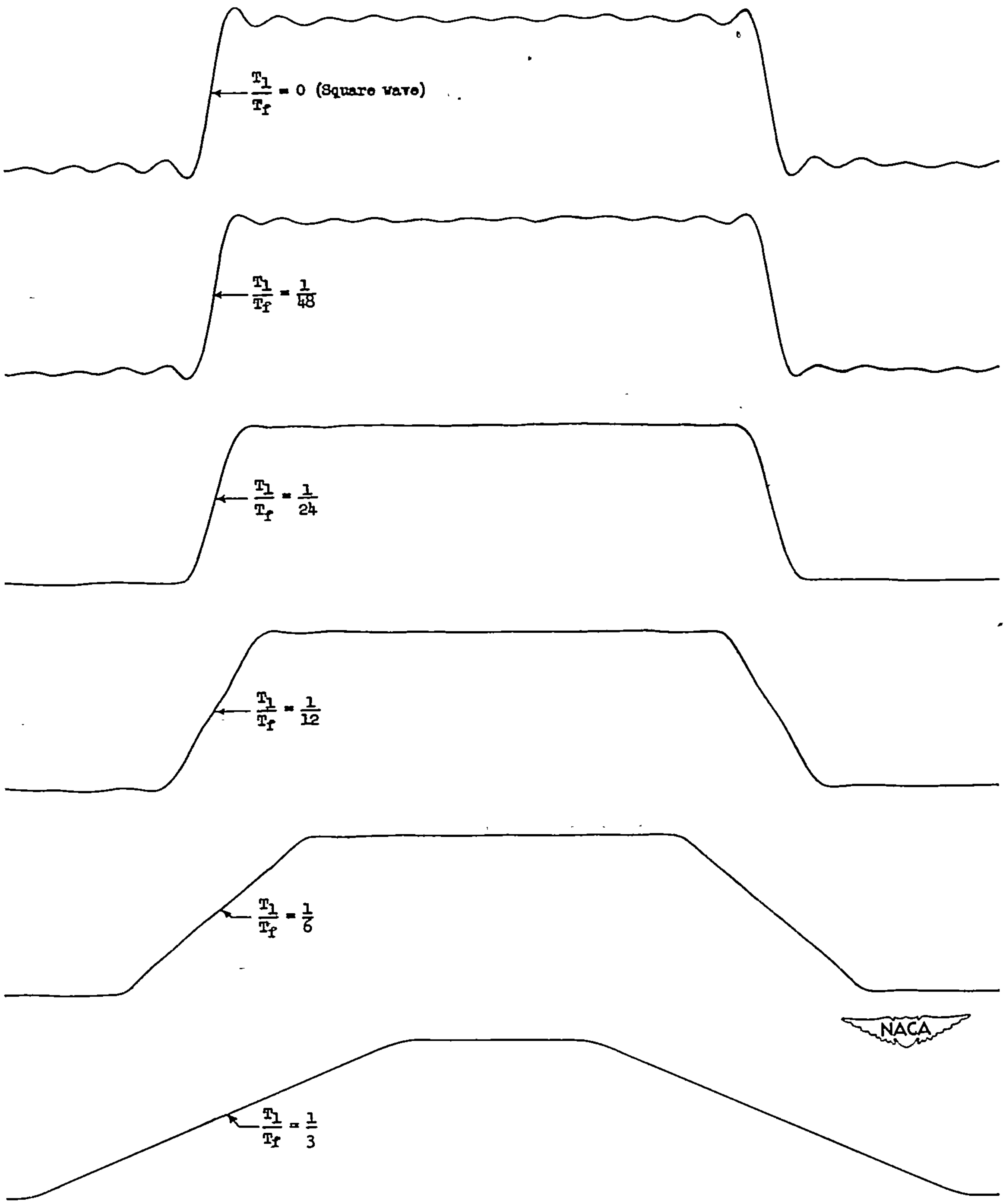


Figure 2.- Input waves representing elevator motion plotted by Fourier synthesizer.

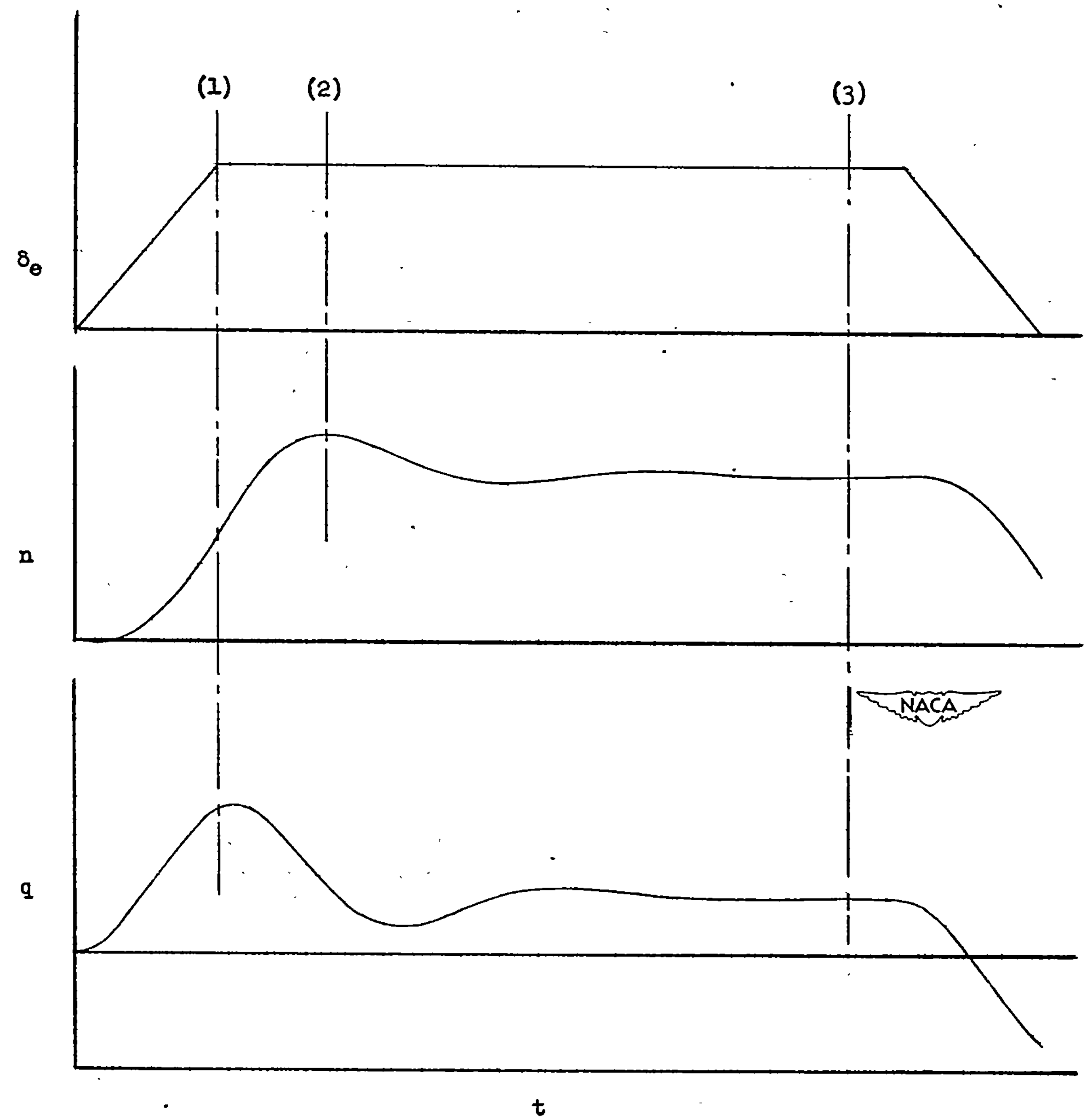
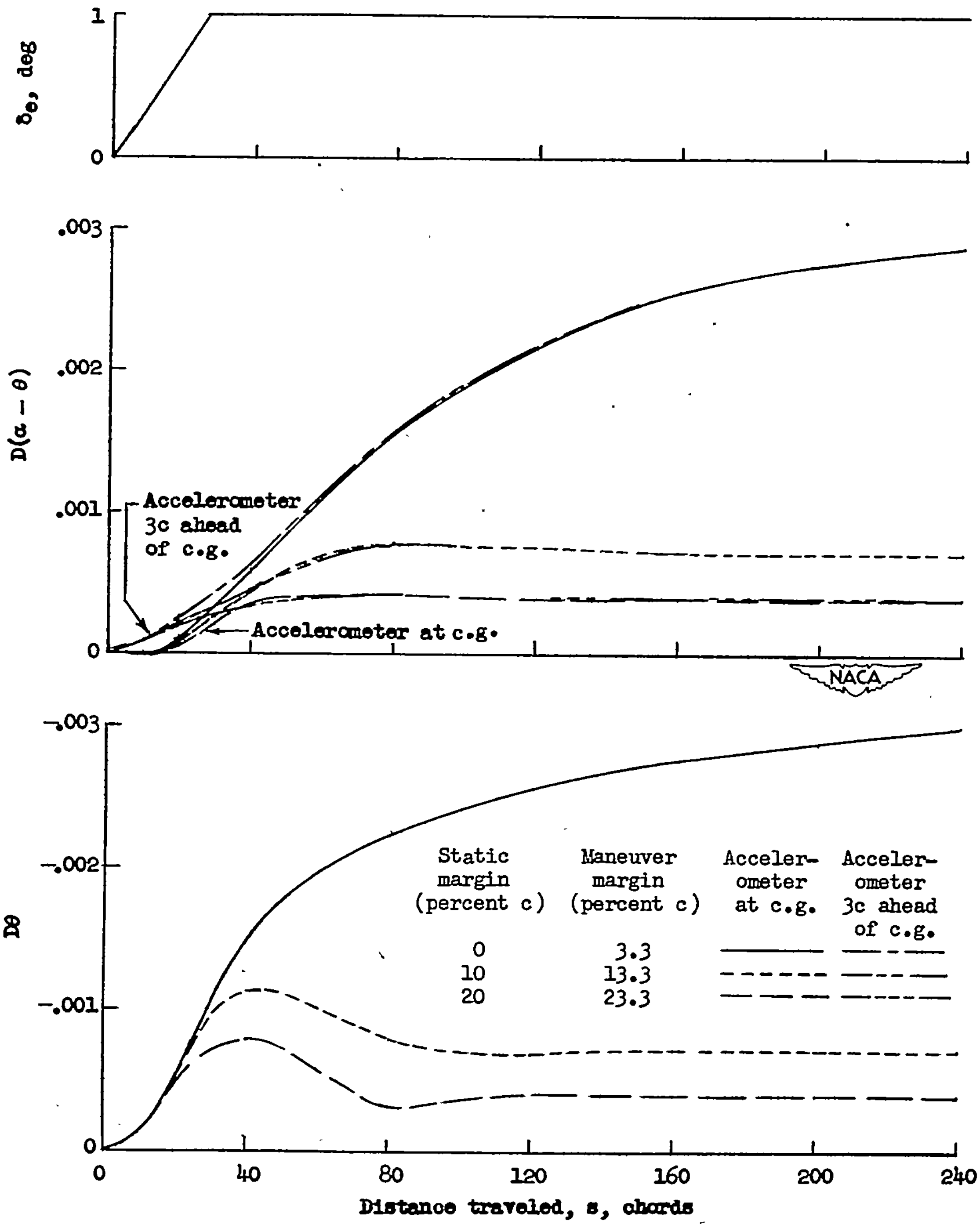
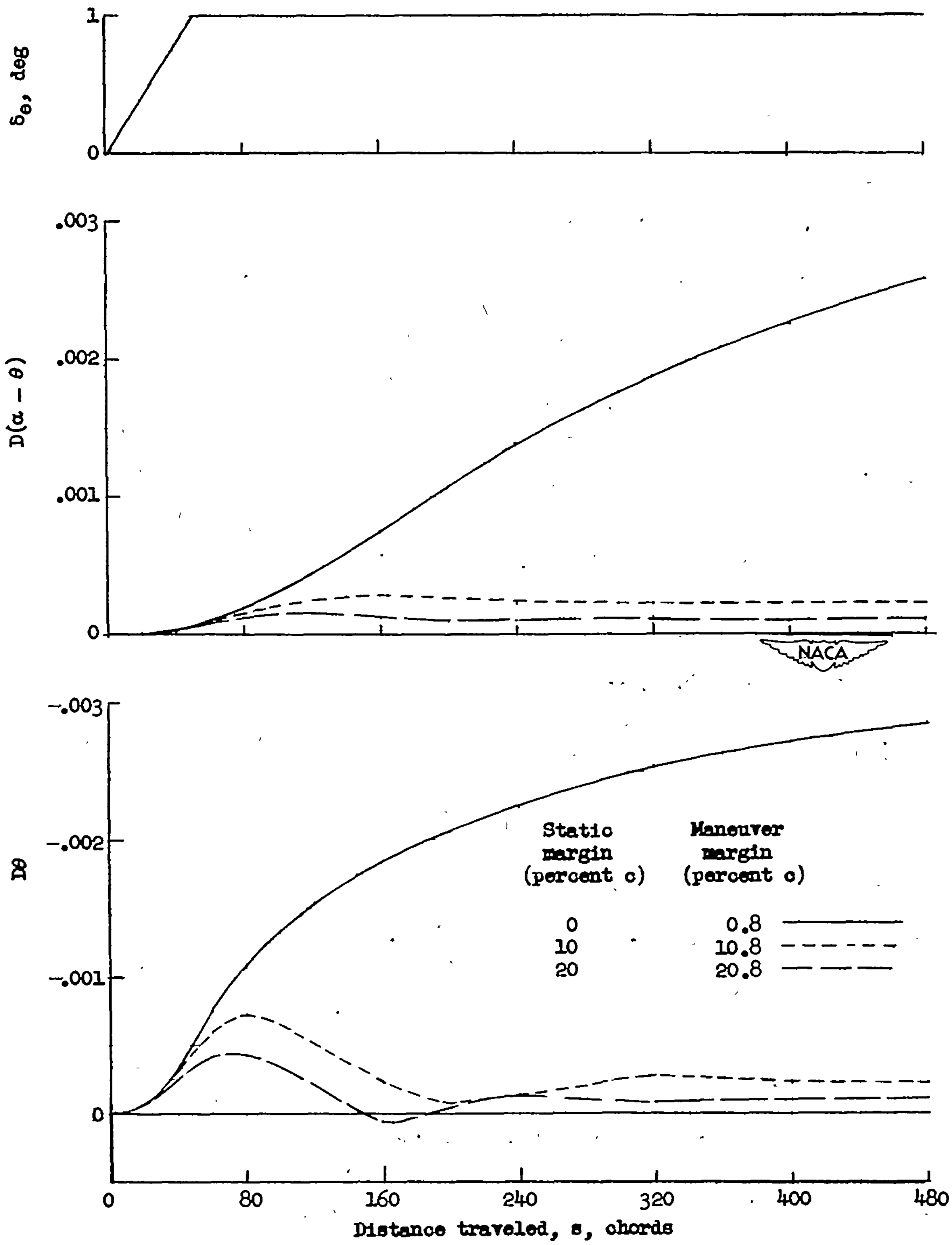


Figure 3.- Typical time history of response of normal acceleration and pitching velocity to elevator motion. Time (1) is the time when the elevator motion stops. Time (2) is the time for maximum normal acceleration. Time (3) is a time when steady conditions have been reached.



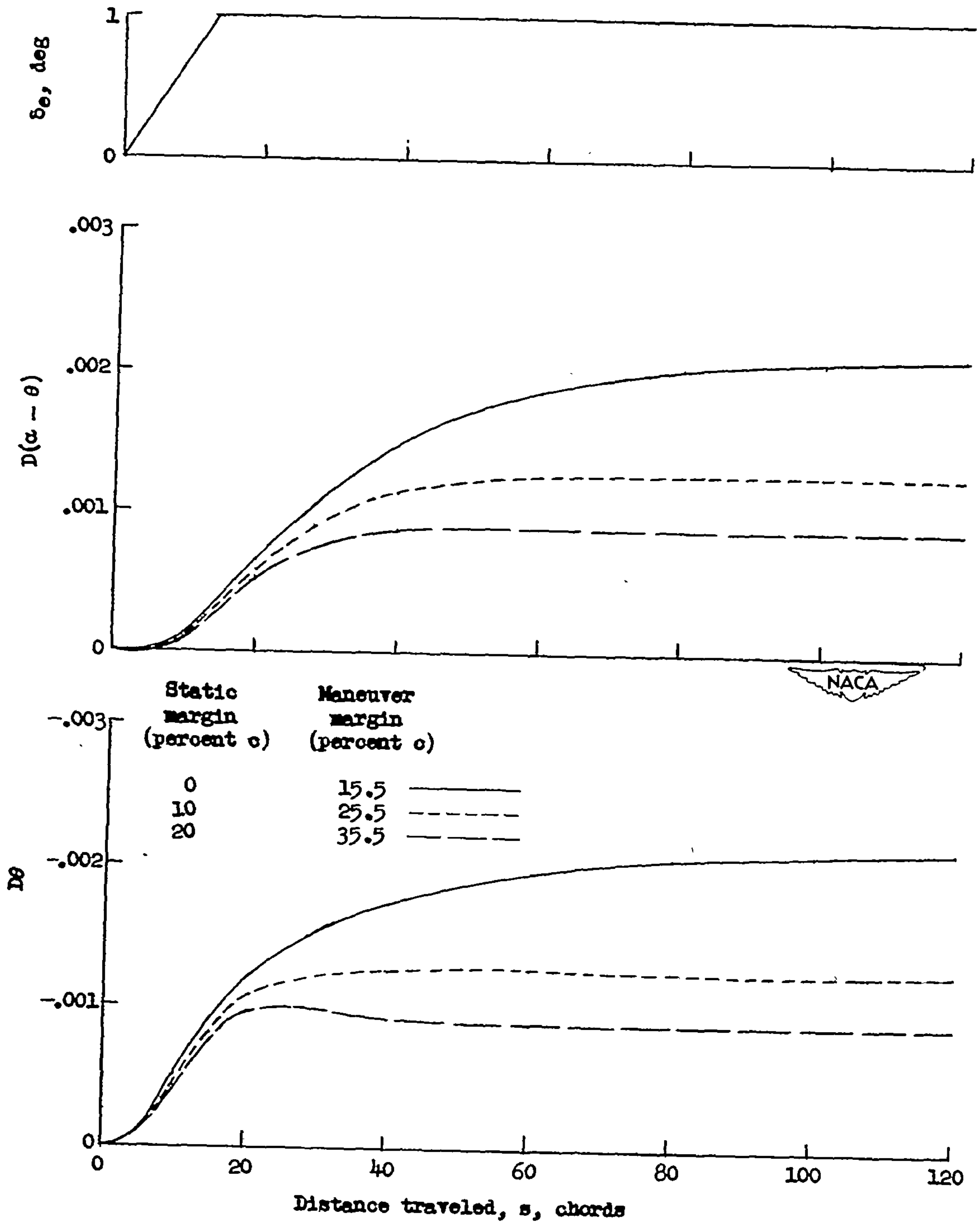
(a) Fighter airplane at sea level.

Figure 4.- Variation of the nondimensional normal-acceleration parameter $D(\alpha - \theta)$ and the nondimensional pitching-velocity parameter $D\theta$ with distance traveled following a rapid elevator movement of 1°.



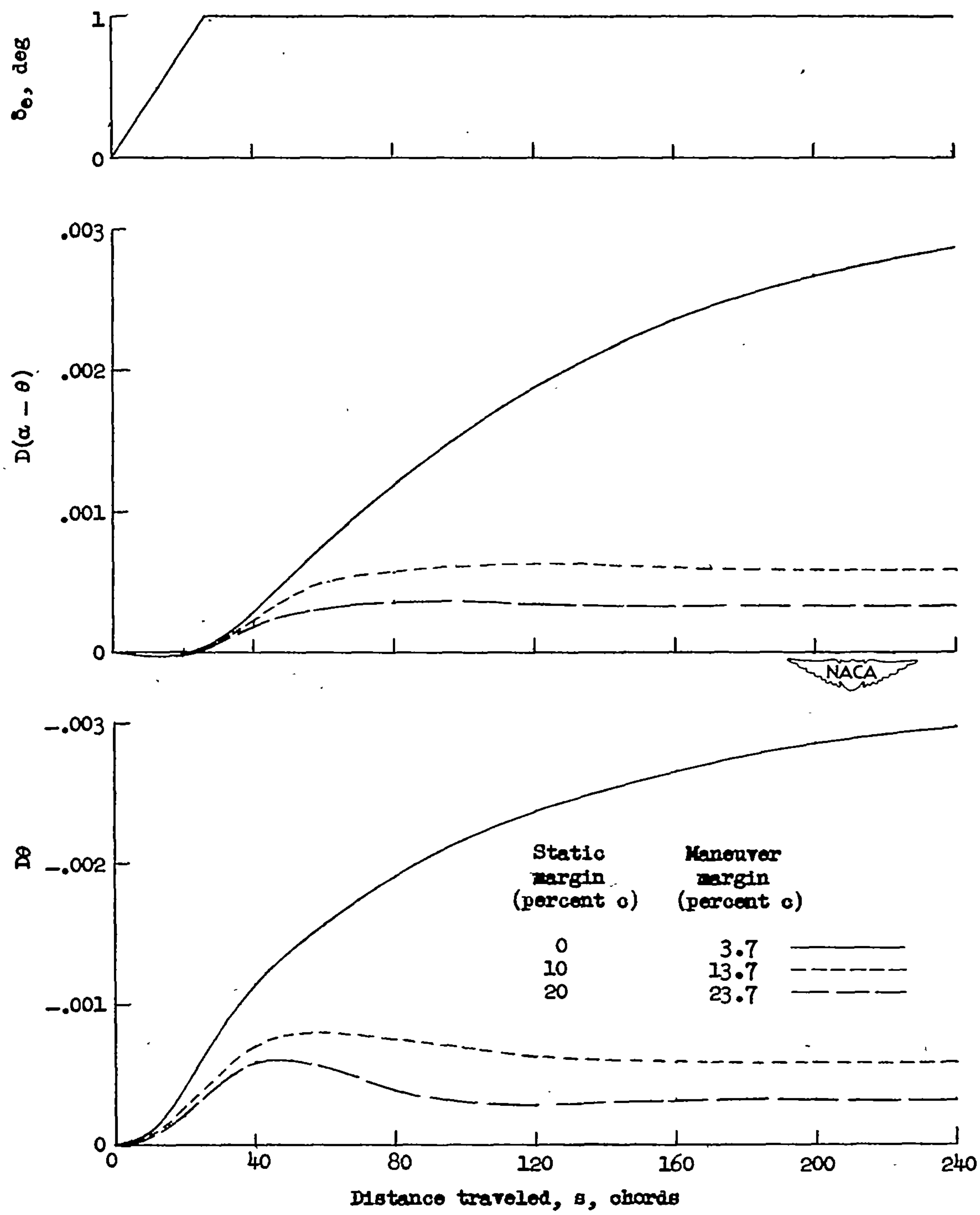
(b) Fighter airplane at 40,000 feet altitude.

Figure 4.- Continued.



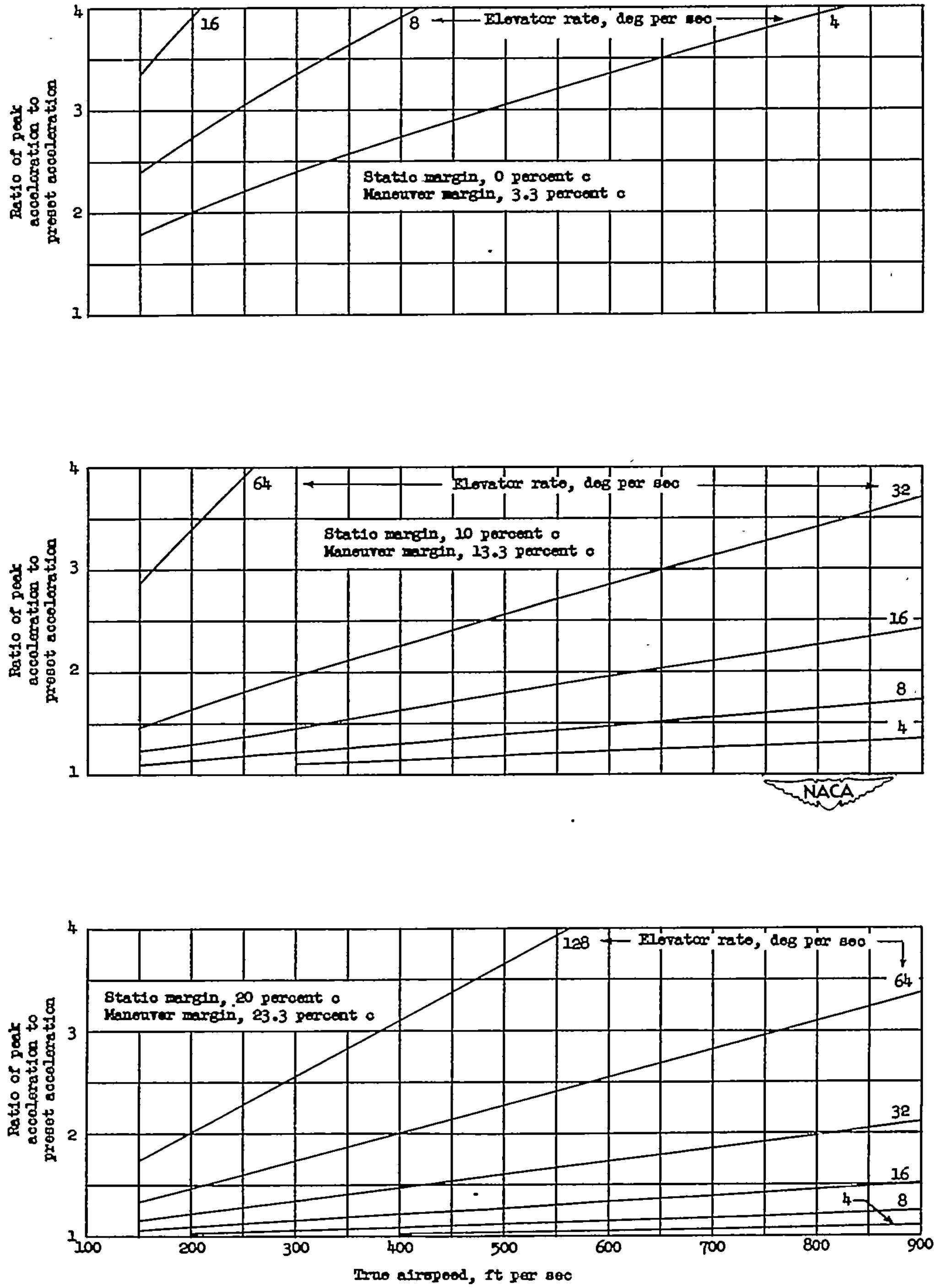
(c) Transport airplane at sea level.

Figure 4.- Continued.



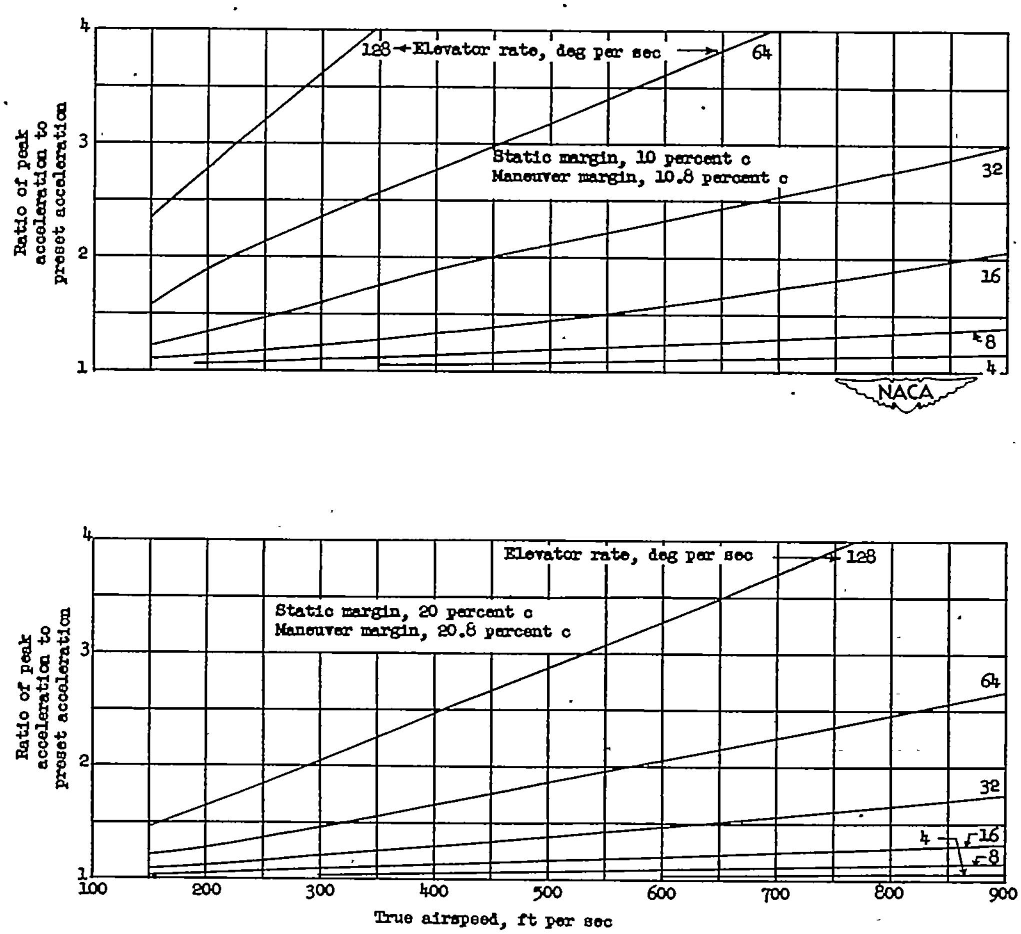
(d) Transport airplane at 40,000 feet altitude.

Figure 4.- Concluded.



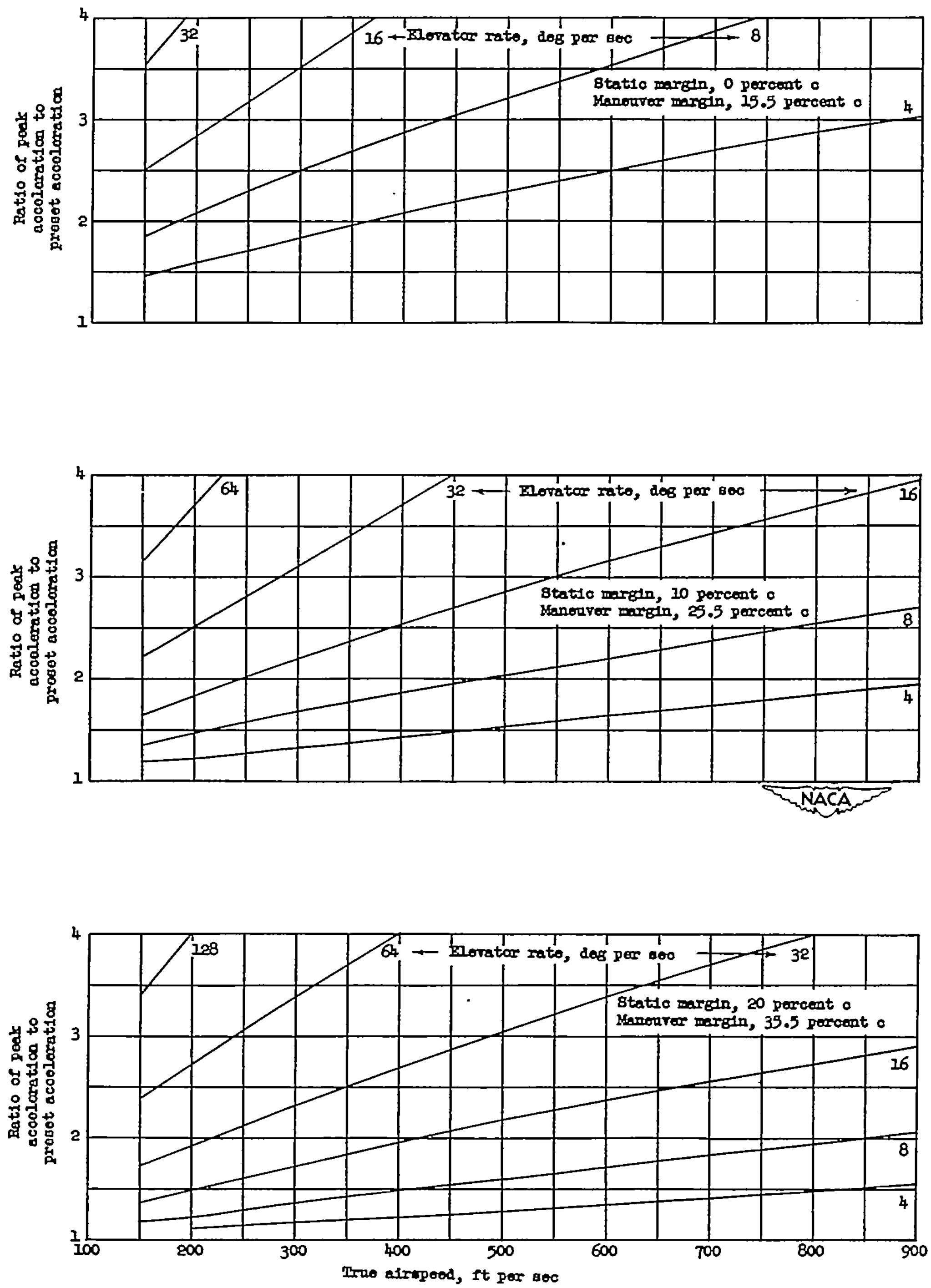
(a) Fighter airplane at sea level, preset acceleration 6g.

Figure 5.- Ratio of peak acceleration to preset acceleration as a function of true airspeed for various values of maximum elevator rate. Acceleration restrictor controlled by accelerometer mounted at center of gravity.



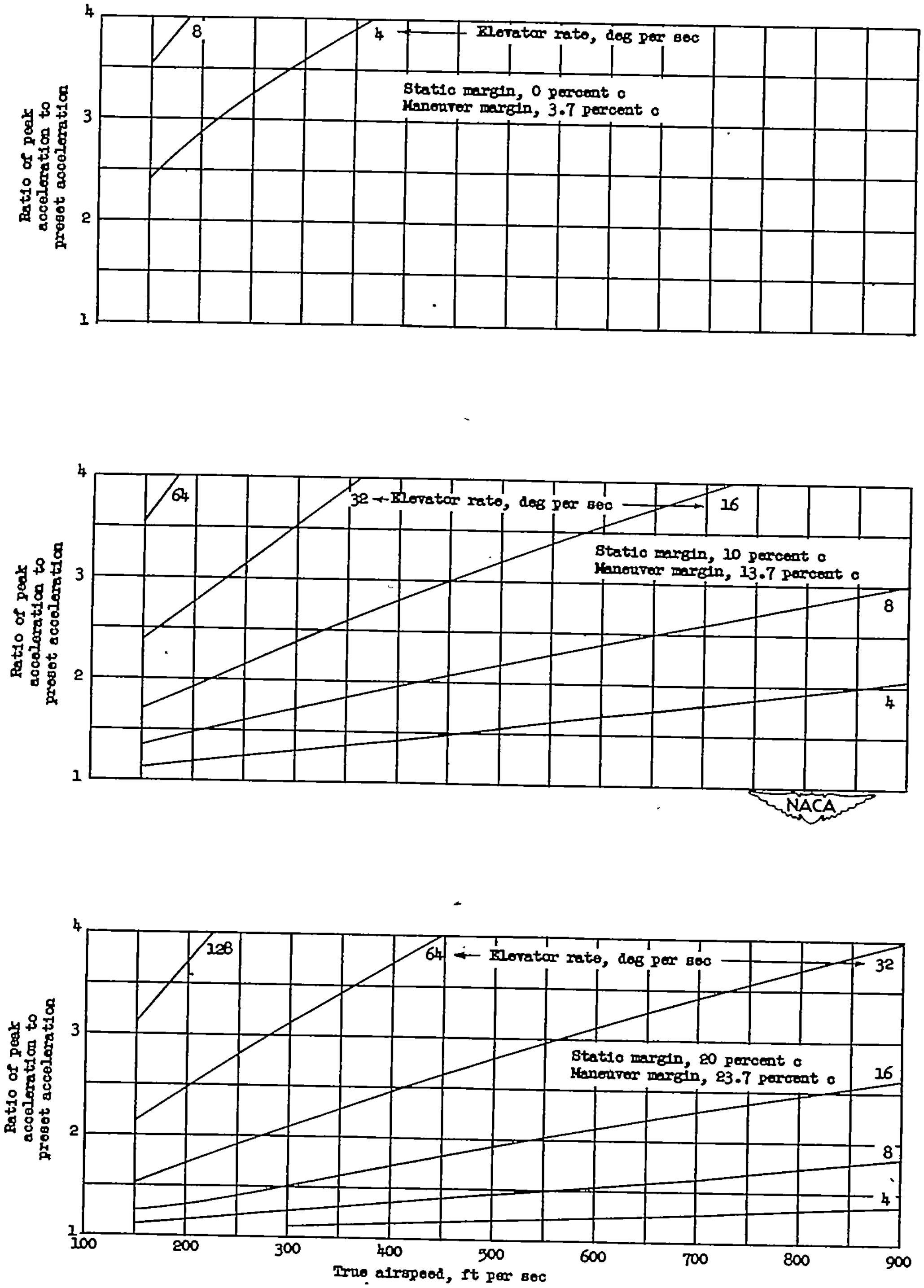
(b) Fighter airplane at 40,000 feet altitude, preset acceleration 6g. (Curves for static margin of zero not shown because all values of elevator rate listed give very large ratios of peak acceleration to preset acceleration.)

Figure 5.- Continued.



(c) Transport airplane at sea level, preset acceleration 2.5g.

Figure 5.- Continued.



(d) Transport airplane at 40,000 feet altitude, preset acceleration 2.5g.

Figure 5.- Concluded.

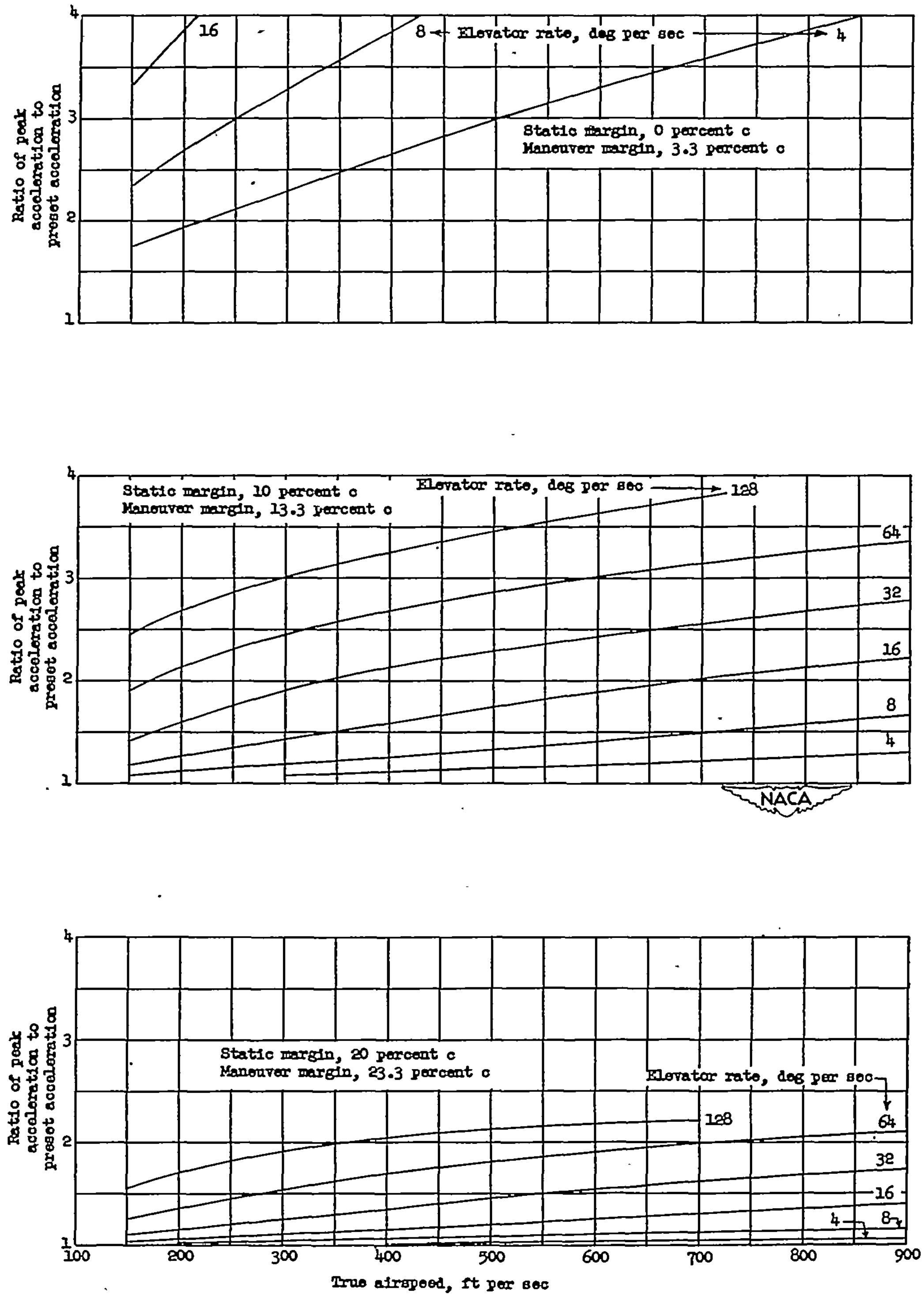
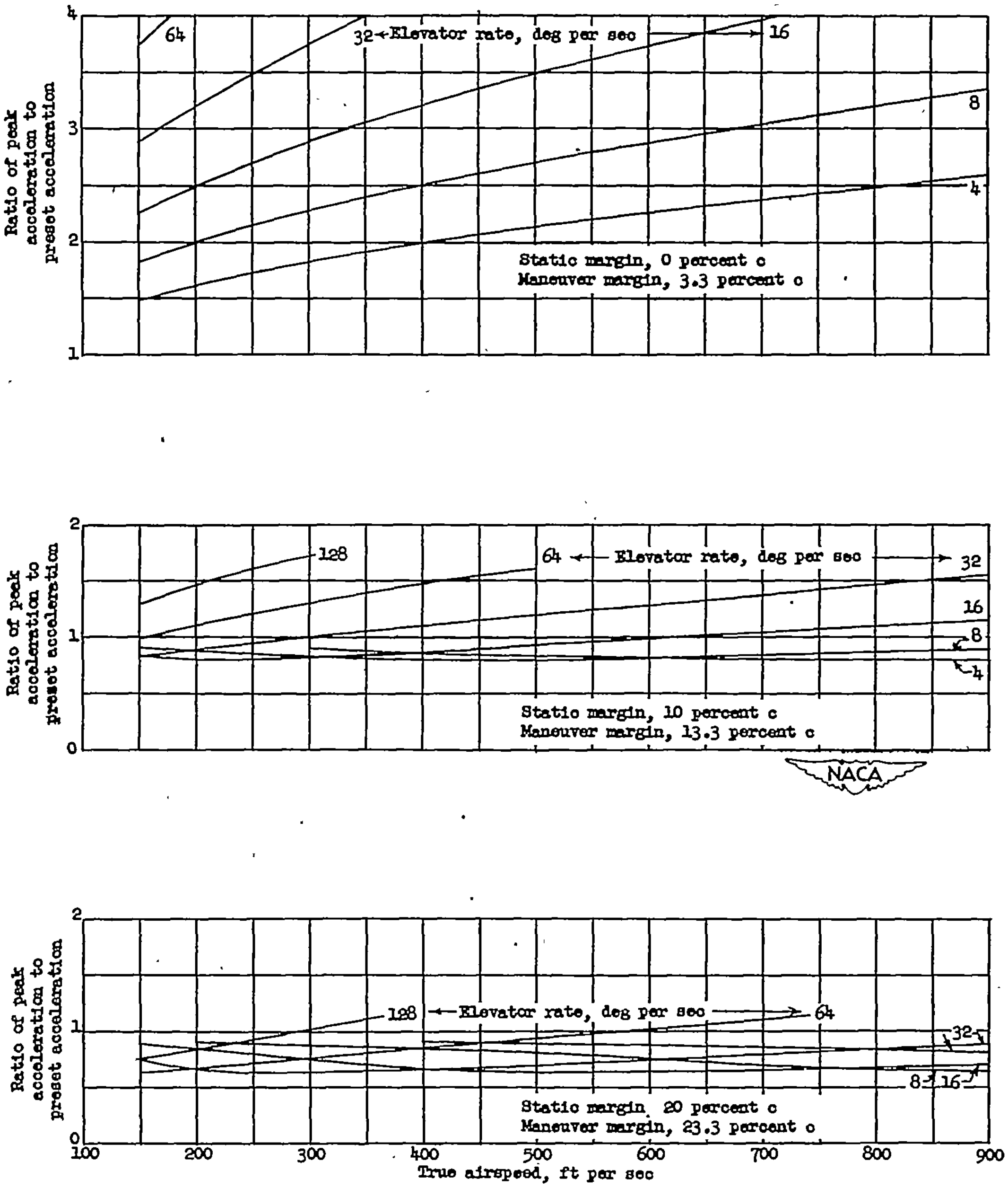
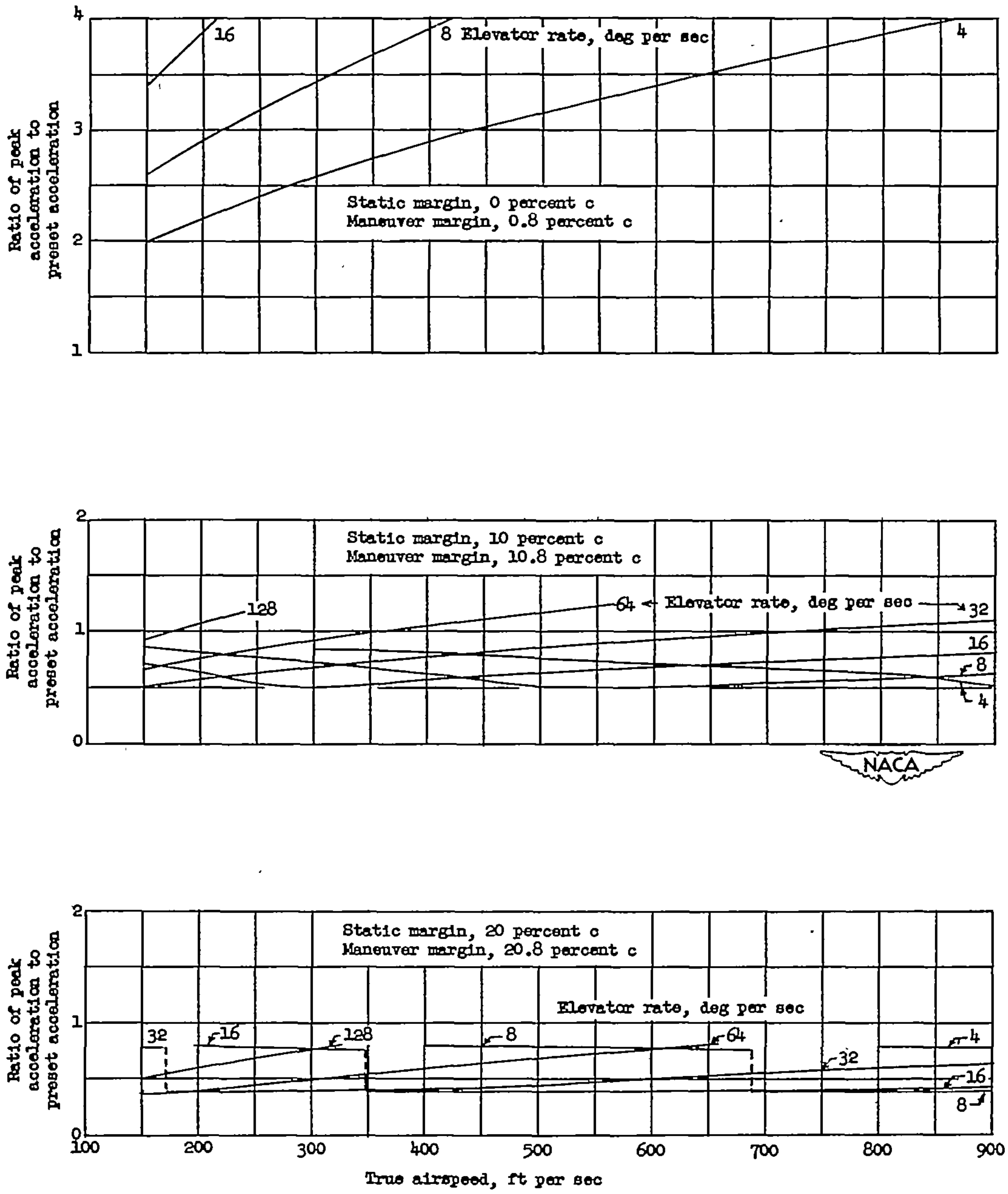


Figure 6.- Ratio of peak acceleration to preset acceleration as a function of true airspeed for various values of maximum elevator rate. Acceleration restrictor controlled by accelerometer mounted 3 chords ahead of center of gravity. Fighter airplane at sea level. Preset acceleration 6g.



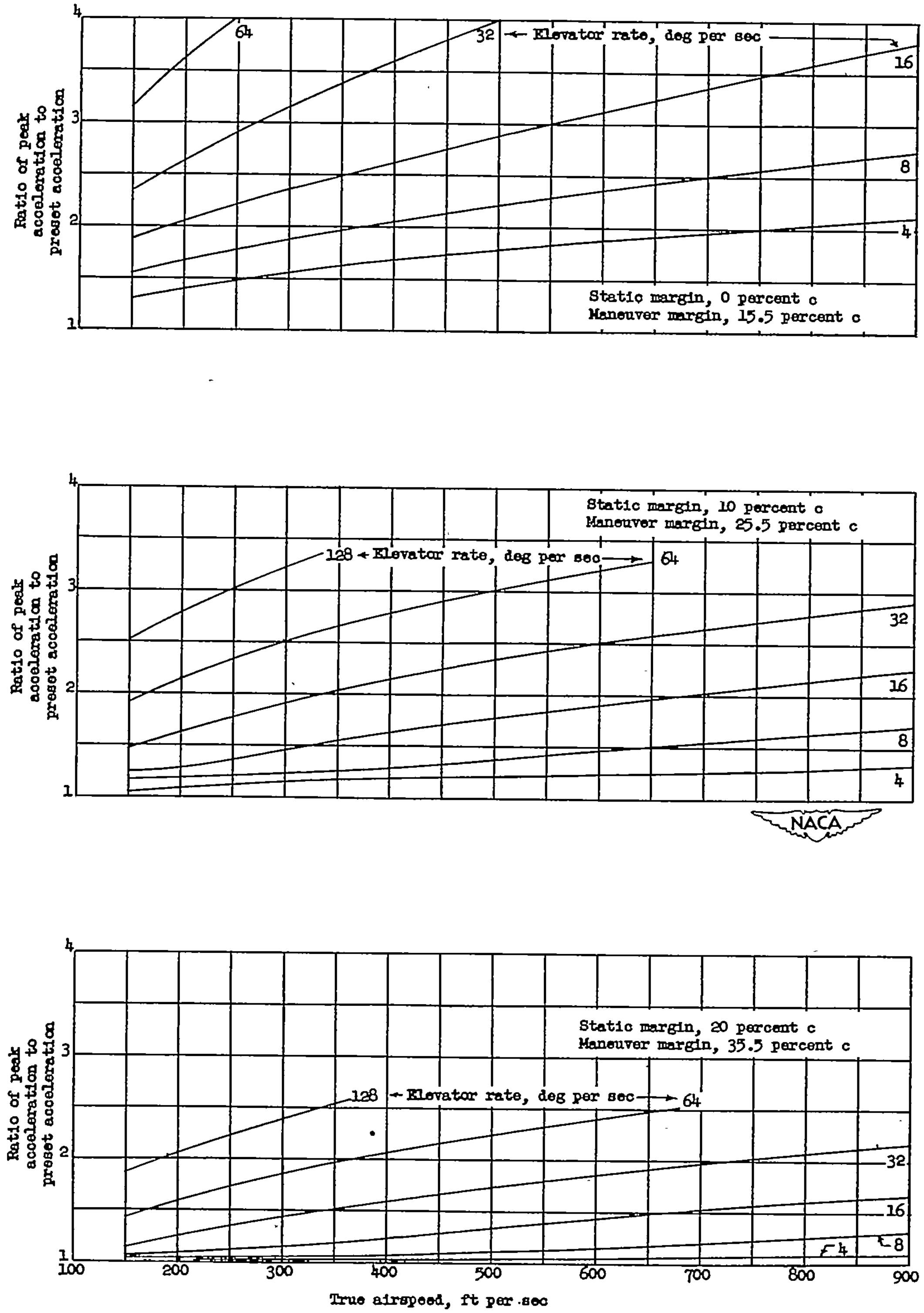
(a) Fighter airplane at sea level, preset acceleration 6g.

Figure 7.- Ratio of peak acceleration to preset acceleration as a function of true airspeed for various values of maximum elevator rate. Acceleration restrictor controlled by a device sensing the quantity Vq/g.



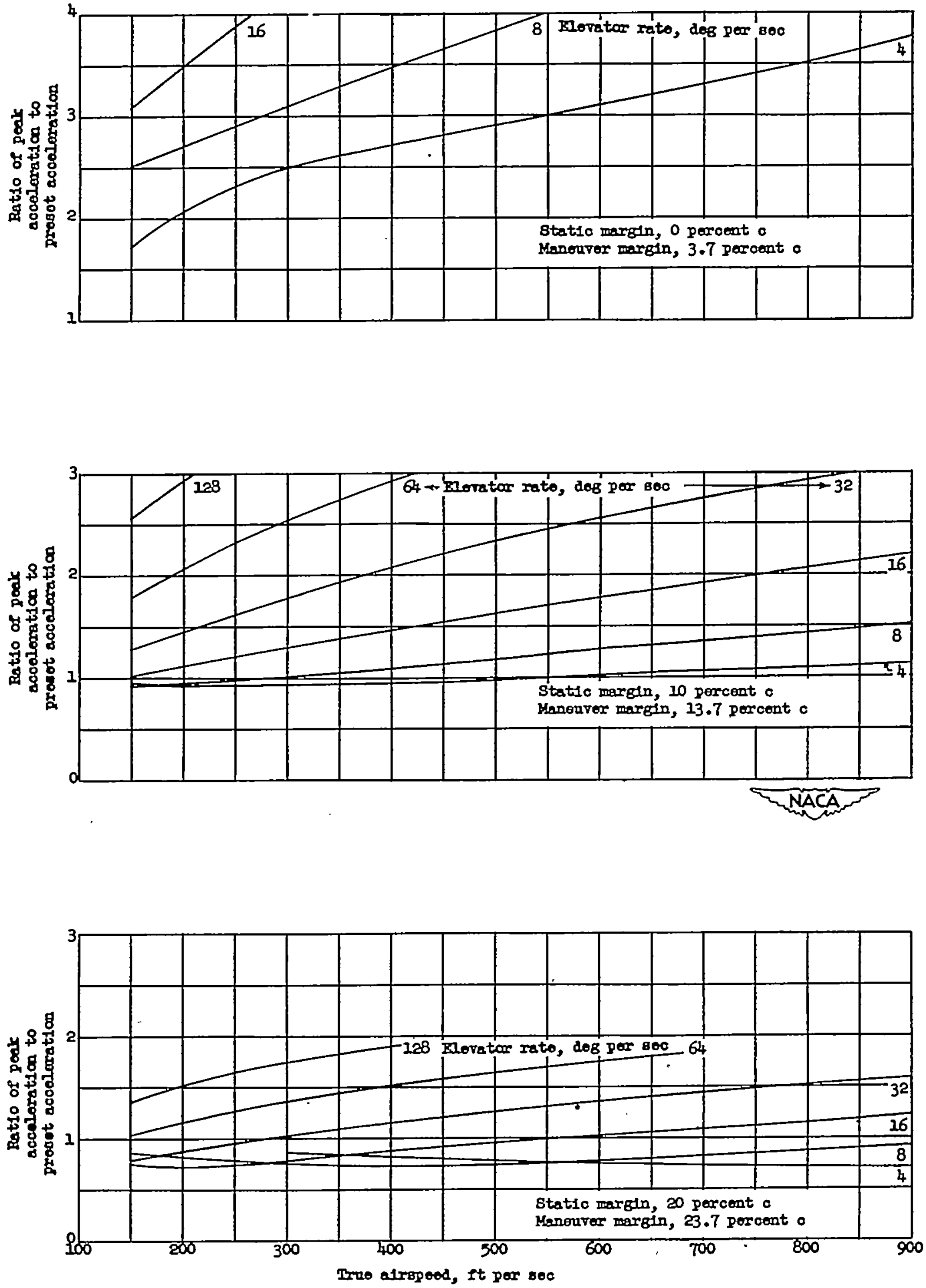
(b) Fighter airplane at 40,000 feet altitude, preset acceleration 6g.

Figure 7.- Continued.



(c) Transport airplane at sea level, preset acceleration 2.5g.

Figure 7.- Continued.



(d) Transport airplane at 40,000 feet altitude, preset acceleration 2.5g.

Figure 7.- Concluded.

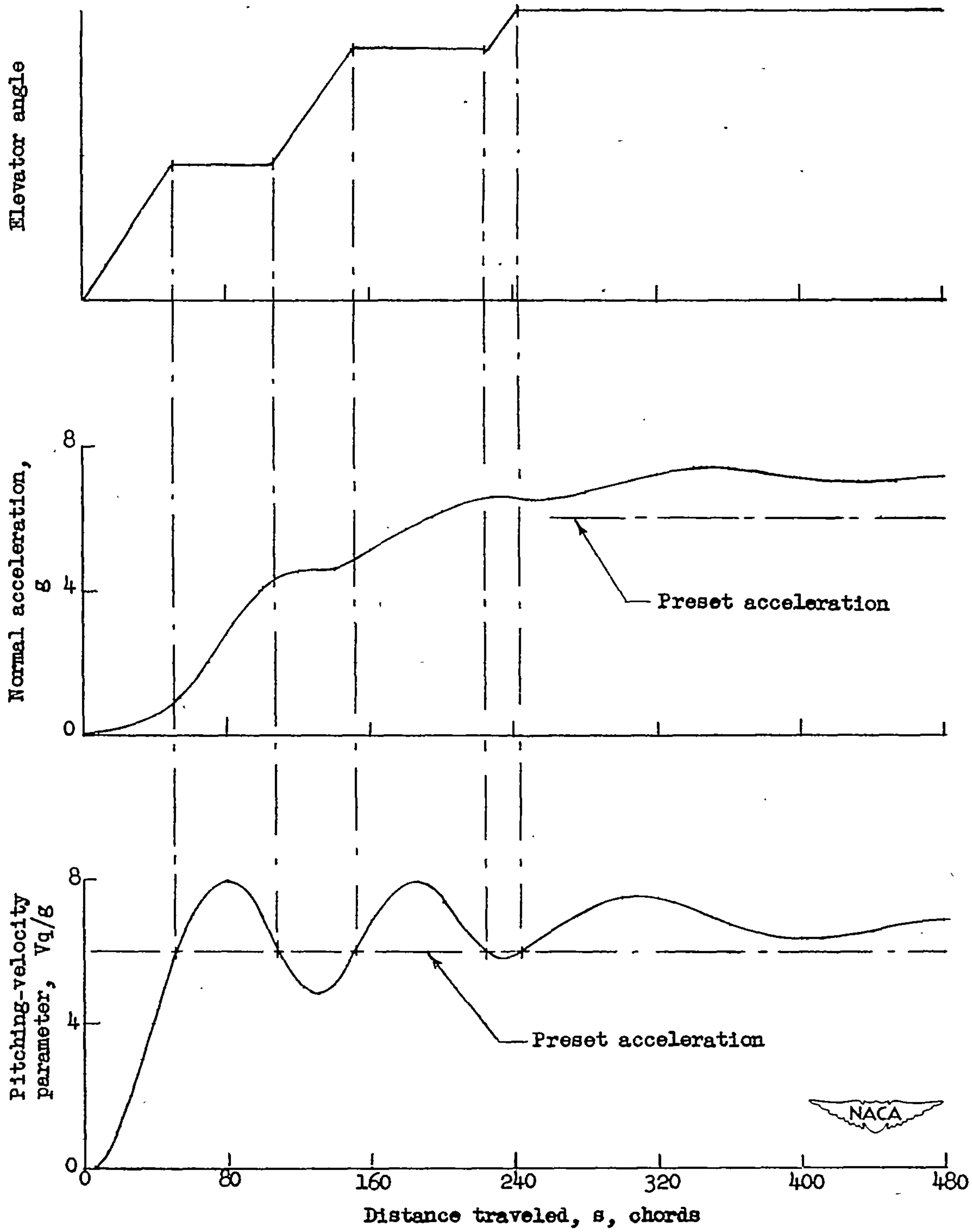


Figure 8.- Typical time history of the action of an acceleration restrictor sensitive to the quantity Vq/g in a case where the elevator motion initially is stopped before the preset acceleration is reached. Preset acceleration 6g. (Elevator-angle scale omitted because the quantitative values of elevator angle depend on the airspeed.)

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